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Influence of Kerr Medium on Entanglement of Cascade-Type Three-Level Atoms and a Bimodal Cavity Field

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Abstract We investigate the influence of Kerr medium on atomic population probability and residual entanglement of the system which consists of cascade-type three-level atoms and a bimodal cavity field filled with Kerr medium. The results show that the period of residual entanglement is shortened and the value of residual entanglement is enhanced by appropriately adjusting the nonlinear Kerr constant. Furthermore, we also study the influence of Kerr medium on entanglement evolution of the two atoms, and find that it decreases the value of entanglement between two atoms.

Keywords Atomic population probability · Residual entanglement · Concurrence · Kerr medium

1 Introduction

Quantum entanglement, which was proposed by Einstein and Schrödinger [1], is one of the essential features of quantum mechanics and plays a key role in quantum information processing [2, 3] and quantum computation [4], such as quantum teleporation [5], superdense coding [6], quantum error correction [7], and so on. Recently, many schemes have been proposed for generation of entangled states with different physical systems, such as cavity QED system [8], optical system [9, 10], and ion trap system [11, 12], etc. It is very important and necessary for studying quantum entanglement in realistic situation. To investigate the entanglement dynamics, we should first choose a proper measure to quantify entanglement. Concurrence is one of the most effective measures, which was proposed by Wootters [13] for measuring entanglement of two qubits pure or mixed states. Moreover,

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due to the relation between concurrence and entanglement of formation, it is interesting for us to know whether the concurrence can be generalized to the larger quantum systems or not, i.e., a three-qubit or even a multi-qubit pure or mixed state system. Lately, Coffman et al. [14] used concurrence to examine three-qubit quantum systems. Furthermore, being a measurement of entanglement, the residual entanglement represents a collective property of all the three-qubit and characterizes an essential three-qubit entanglement [15–17]. Several researchers [18–20] have proposed physically motivated postulates to characterize three-body entanglement measures. Li et al. [21] researched the three-body entanglement in Tavis-Cummings model by calculating Akhtarshenas's concurrences. Gao et al. [22] considered the effect of cavity decay on entanglement between Ladder-Type three-level atoms and a two-mode cavity field with the presence of decoherence effect, and they found that the residual entanglement is less than unity. In addition, the influence of Kerr medium on entanglement dynamics of many different systems have been reported by different measuring methods [23–25]. Ma et al. [26] investigated the properties of quantum entanglement in the two-photon Tavis-Cummings model with a Kerr nonlinearity in terms of quantum information entropy theory. Particularly, tripartite entanglement dynamics of system with a Kerr nonlinearity has also been widely researched and characterized [27]. For a specific value of the Kerr-like medium, they found that the entanglement may terminate suddenly in a finite time. In this paper, we investigate the effect of Kerr medium on entanglement of the cascade-type three-level atoms with the bimodal cavity field. Our results show that Kerr medium can shorten remarkably the period of the atomic population probability and the residual entanglement under ideal situation. In the meantime, Kerr medium can increase the peak value of the atomic population probability and the residual entanglement when the cavity decay is considered. Furthermore, we also find that the entanglement between the two atoms decreases notably when the bimodal cavity field is filled with Kerr medium. The investigation of this work could provide a indispensable theoretics foundation for the future experiment.

The rest of the paper is organized as follows. In Sect. 2, we briefly introduce a physical model which describes the interaction between a cascade-type three-level atom and a bimodal cavity field filled with Kerr medium. In Sect. 3, we calculate and discuss the influence of Kerr medium on residual entanglement of the first atom and the bimodal cavity field. In Sect. 4, we study the influence of Kerr medium on residual entanglement when the cavity decay is considered. In Sect. 5, we investigate the influence of Kerr medium on the entanglement between two atoms. Finally, a brief conclusion is given in Sect. 6.

2 The Model

Consider a cascade-type three-level atom interacting with the bimodal cavity field. Assume that the transitions $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$ of the atom interact with the cavity modes a and b, respectively, as shown in Fig. 1. The interaction Hamiltonian under the interaction picture is

$$H = \hbar(\lambda_1|e\rangle\langle g|\hat{a}e^{i\Delta_1 t} + \lambda_2|f\rangle\langle e|\hat{b}e^{-i\Delta_2 t} + \text{H.c.}), \tag{1}$$

where a ($a^\dagger$) and b ($b^\dagger$) denote the annihilation (creation) operators for the bimodal mode cavity, respectively; λ_1 (λ_2) is the atom-cavity coupling constant. $\Delta_1 = \omega_{eg} - \omega_1$ ($\Delta_2 = \omega_{fe} - \omega_2$) is the detuning, where ω_{eg} (ω_{fe}) is the atomic transition frequency, ω_1 (ω_2) is the frequency of the cavity mode.

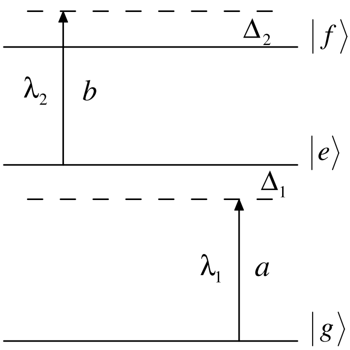

Fig. 1 Three-level atomic configuration, whose levels are denoted by $|f\rangle$, $|e\rangle$, and $|g\rangle$. Here, λ_1 and λ_2 denote the atom-cavity coupling constants, Δ_1 and Δ_2 are the detunings

We take $\lambda_1 = \lambda_2 = \lambda$, $\Delta_1 = -\Delta_2 = \Delta$, $\hbar = 1$ and consider that the cavity field is filled with Kerr medium. Under the adiabatical approximation and the large detuning condition, the effective Hamiltonian of describing the system can be written as

$$H_{\rm eff} = -\frac{\lambda^2}{\Delta}(|g\rangle\langle g|\hat{a}^\dagger\hat{a} + |f\rangle\langle f|\hat{b}\hat{b}^\dagger + |g\rangle\langle f|\hat{a}^\dagger\hat{b}^\dagger + |f\rangle\langle g|\hat{a}\hat{b}) + \chi(\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}), \tag{2}$$

where χ is the nonlinear coupling constant of Kerr medium.

3 Residual Entanglement of an Atom and a Bimodal Cavity

We send a three-level atom into the bimodal cavity. Assume that the atom and the bimodal cavity field are initially in the states $|g\rangle_1$ and $|11\rangle_{ab}$, respectively. The time evolution of the state of the atom-cavity system, governed by the effective Hamiltonian (2), is given by

$$|g11\rangle_{1ab} \to x_1(t_1)|g11\rangle_{1ab} + y_1(t_1)|f00\rangle_{1ab}, \tag{3}$$

where

$$x_1(t_1) = e^{iv_1t_1}\left[\cos(\Omega_1 t_1) - \frac{(i\chi/2)}{\Omega_1}\sin(\Omega_1 t_1)\right], \tag{4}$$

$$y_1(t_1) = \frac{i\lambda^2}{\Omega_1\Delta}e^{iv_1t_1}\sin(\Omega_1 t_1), \tag{5}$$

with

$$v_1 = \lambda^2/\Delta - \chi/2, \tag{6}$$

$$\Omega_1 = [(\lambda^2/\Delta)^2 - (i\chi/2)^2]^{1/2}. \tag{7}$$

Using the calculation given by (4), we may easily obtain the population probability that the three-level atom is found in the state $|g\rangle_1$, as

$$P_g = |x_1(t_1)|^2. \tag{8}$$

According to (8), we drew the time evolution of the atomic population probability P_g for different nonlinear Kerr constant χ, as shown in Fig. 2(a). Comparing the solid line with the

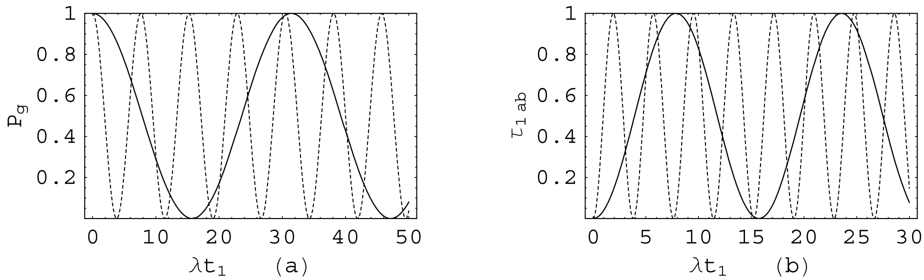

Fig. 2 (**a**) The time evolution of the atomic population probability. (**b**) The time evolution of the residual entanglement between the atom and the bimodal cavity field. Here, the *dashed line* stands for $\chi = 0.8\lambda$, the *solid line* stands for $\chi = 0$, and $\Delta = 10\lambda$

dashed line, we find that the period of P_g with Kerr medium shortens rapidly. The higher the value of χ is, the shorter the period of P_g is. This means that Kerr medium can shorten the period of P_g and enhance the mean value of P_g availably.

In the following, we utilize the concurrence $C(\rho)$, which is first defined by Wootters and is widely accepted for any two qubits case [13], to quantify the degree of entanglement. The concurrence varies from $C = 0$ for a unentangled state to $C = 1$ for a maximally entangled state. The corresponding concurrence of the density matrix ρ can be written as

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{9}$$

where ρ is the reduced density matrix of qubits, λ_1, λ_2, λ_3, and λ_4 are four non-negative square roots of the eigenvalues of the non-Hermitian matrix $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order. ρ^* is the complex conjugation of ρ in the standard basis, and σ_y is the Pauli matrix. The entanglement of three-partite pure state can be measured by residual entanglement, which is defined as [15–17]

$$\tau_{ABC} = C_{A(BC)} - C_{AB}^2 - C_{AC}^2, \tag{10}$$

where the tangle $C_{A(BC)}$ between a subsystem A and the rest of the global system (denoted as B, C) is represented as

$$C_{A(BC)} = 2(1 - \mathrm{tr}\rho_{\mathrm{A}}^2), \tag{11}$$

and C_{AB} (C_{AC}) is the well-known concurrence for entanglement measurement of qubits A and B (A and C) [13]. From (3), (9), (10) and (11), we can obtain residual entanglement of the atom and the bimodal cavity through simple calculation, as below

$$\tau_{1ab} = 4|x_1(t_1)|^2|y_1(t_1)|^2. \tag{12}$$

As shown in Fig. 2(b), we plotted the evolutions of residual entanglement τ_{1ab} following the time t for various values of χ. From Fig. 2(b), we can directly see that the period of τ_{1ab} with Kerr medium is shorter than that without Kerr medium, which means that the mean value of τ_{1ab} with Kerr medium is higher than that without Kerr medium.

4 Residual Entanglement of System When the Cavity Decay Is Considered

Next, we study the influence of Kerr medium on residual entanglement of the atom and a bimodal cavity when the cavity decay is considered. In this case, the effective Hamiltonian of the system is written as

$$\begin{aligned} H_{\text{eff}}^{'} = & -\frac{\lambda^2}{\Delta}(|g\rangle\langle g|\hat{a}^\dagger\hat{a} + |f\rangle\langle f|\hat{b}\hat{b}^\dagger + |g\rangle\langle f|\hat{a}^\dagger\hat{b}^\dagger + |f\rangle\langle g|\hat{a}\hat{b}) \\ & - i\kappa(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}) + \chi(\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}), \end{aligned} \tag{13}$$

where κ denotes the rate of decay of the cavity modes a and b. The time evolution of the atom-cavity system under the effective Hamiltonian of (13) can be written as

$$|g11\rangle_{1ab} \rightarrow x_2(t_2)|g11\rangle_{1ab} + y_2(t_2)|f00\rangle_{1ab}, \tag{14}$$

where

$$x_2(t_2) = e^{iv_2t_2}\left[\cos(\Omega_2t_2) - \frac{\kappa + (i\chi/2)}{\Omega_2}\sin(\Omega_2t_2)\right], \tag{15}$$

$$y_2(t_2) = \frac{i\lambda^2}{\Omega_2\Delta}e^{iv_2t_2}\sin(\Omega_2t_2), \tag{16}$$

with

$$v_2 = \lambda^2/\Delta + i\kappa - \chi/2, \tag{17}$$

$$\Omega_2 = [(\lambda^2/\Delta)^2 - (\kappa + (i\chi/2))^2]^{1/2}. \tag{18}$$

Similarly, we obtain the atomic population probability and the residual entanglement, as

$$P_g^{'} = |x_2(t_2)|^2, \tag{19}$$

$$\tau_{1ab}^{'} = 4|x_2(t_2)|^2|y_2(t_2)|^2. \tag{20}$$

We plotted the evolution of the atomic population probability P_g' with the time for different nonlinear Kerr constant χ when the cavity decay κ is fixed, as shown in Fig. 3(a). The dashed line and the solid line stand for P_g' with and without Kerr medium, respectively. In the presence of Kerr medium, the period of P_g' is shortened and the oscillation amplitude of P_g' is strengthened. So we can see that the mean value of P_g' is enhanced obviously. Furthermore, it is worth noting that P_g' falls to zero following the increment of time t in the absence of Kerr medium. However, P_g' reaches a fixed value following the increment of time t in the presence of Kerr medium. From the above analysis, we can obtain the following results of the influence of Kerr medium on P_g': (i) the value of P_g' is rapidly enhanced; (ii) the period of P_g' is shortened; (iii) P_g' reaches a fixed value following the evolution of the time t.

Figure 3(b) denotes the time evolution of the residual entanglement between the atom and the bimodal cavity field. For a fixed κ, the solid line denotes τ_{1ab}' without Kerr medium, the dotted line and the dashed line represent τ_{1ab}' with different nonlinear Kerr constants. Comparing the solid line with the dashed line, we can directly see that the maximal value of τ_{1ab}' without Kerr medium merely reaches 0.2, its period largens and τ_{1ab}' disappears quickly due to the cavity decay. However, the maximal value of τ_{1ab}' with Kerr medium

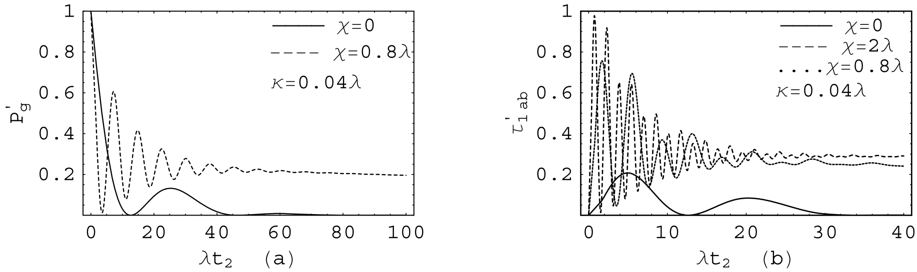

Fig. 3 (**a**) Time evolution of the atomic population probability. (**b**) Time evolution of the residual entanglement between the atom and the bimodal cavity field. Here, we have set $\Delta = 10\lambda$

largens evidently and tends to be stable entanglement with the evolution of time, and its period shortens rapidly. The above analysis shows that: (i) τ'_{1ab} is sensitive to nonlinear Kerr medium; (ii) the appending of Kerr medium increases the maximal value of τ'_{1ab} and shortens the period of τ'_{1ab}; (iii) τ'_{1ab} with Kerr medium reaches a stationary entanglement with the evolution of time, which means that the nonlinear Kerr medium can prevent the death of τ'_{1ab} from occurring; (iv) the adding of Kerr medium strengthens the interaction intensity of the atom with the cavity field and restrains the decoherence effect. Furthermore, comparing the dashed line with the dotted line, we can see from Fig. 3(b) that different nonlinear Kerr constants correspond to different τ'_{1ab}, the larger the nonlinear Kerr constant is, (i) the bigger the value of τ'_{1ab} is; (ii) the smaller the period of τ'_{1ab} is; (iii) the more strongly the τ'_{1ab} oscillates. Moreover, both the two curves turn up two stationary entanglements with the evolution of time. The stable value of τ'_{1ab} and the peak value number of τ'_{1ab} increase following the increment of nonlinear Kerr constant, showing that the total mean value of τ'_{1ab} increases with the increasing of nonlinear Kerr constant. Therefore, it is easy for us to obtain optimal entanglement by adjusting nonlinear Kerr medium reasonably.

5 Concurrence between Two Atoms

We now let another identical atom pass through the bimodal cavity, because of the quantum field inside the cavity, the two atoms become entangled. If we choose $|gg11\rangle_{12ab}$ as the initial state, the time evolution of the two atoms with the bimodal field is given by

$$|gg11\rangle_{12ab} \longrightarrow x_2(t_2)x_2(t_2')|gg11\rangle_{12ab} + x_2(t_2)y_2(t_2')|gf00\rangle_{12ab} + y_2(t_2)|fg00\rangle_{12ab}, \quad (21)$$

where $x_2(t_2')$ $(y_2(t_2'))$ has the same form with $x_2(t_2)$ $(y_2(t_2))$. The subscripts 1 and 2 represent the two identical three-level atoms, and the subscripts a and b denote the two modes of the cavity field. According to (9) and (21), we calculate the concurrence between the two atoms, as

$$C_{12} = 2|x_2(t_2)y_2(t_2)y_2(t_2')|. \quad (22)$$

We plotted the evolution of the concurrence between two atoms with time λt_2, $\lambda t_2'$, as shown in Fig. 4. Compared Fig. 4(a) with 4(b), we find that the peak value of the concurrence decreases fleetly when the nonlinear Kerr constant is appended, which means that concurrence is sensitive to nonlinear Kerr effect. This shows that the presence of nonlinear Kerr effect reduces the entanglement of two atoms. Therefore, we can appropriately select

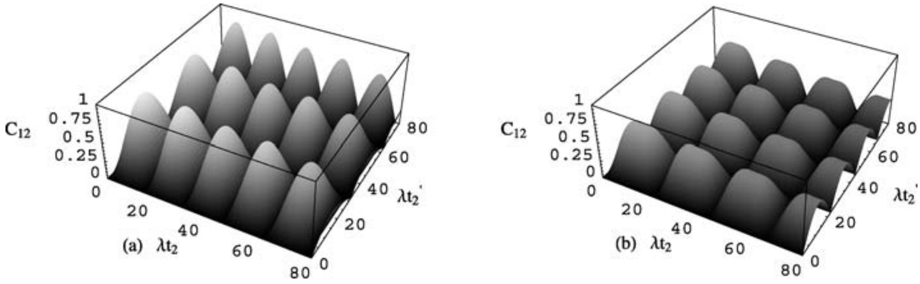

Fig. 4 (**a**) The time evolution of concurrence between two atoms for $\chi = 0$. (**b**) The time evolution of concurrence between two atoms for $\chi = 0.2\lambda$. Here, $\Delta = 10\lambda$, $\kappa = 0.002\lambda$

nonlinear Kerr medium to obtain the optimal value of concurrence, providing a theoretical foundation for future experimental work.

6 Conclusions

We have studied the influence of Kerr medium on the residual entanglement and the atomic population probability of the system that consists of cascade-type three-level atoms and a bimodal cavity field filled with Kerr medium. The results show that: (i) the residual entanglement of the system is very sensitive to nonlinear Kerr medium; (ii) the nonlinear Kerr medium can shorten rapidly the period of the residual entanglement and increase the mean value of residual entanglement of system at ideal situation; (iii) the nonlinear Kerr medium can notably enhance the residual entanglement of the system when the cavity decay is considered. The oscillational amplitude of the residual entanglement increases with increasing the nonlinear Kerr constant, and the residual entanglement trends to be a stationary value following the time evolution. Therefore, we may properly regulate the nonlinear Kerr constant to obtain the maximal value of residual entanglement. Furthermore, Kerr medium also has a negative effect on the concurrence of two atoms, namely, the concurrence decreases when the nonlinear Kerr medium is added. Thus we can appropriately utilize Kerr medium to obtain the optimal entanglement between the atoms.

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